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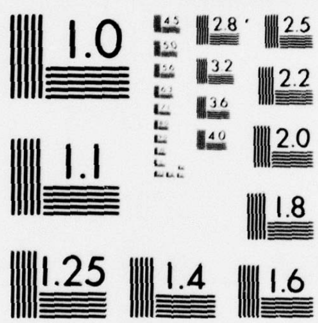

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On Approaches to Robust Detection for HF Communications

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Joel M. Morris

Systems Integration and Instrumentation Branch
Communications Sciences Division

January 25, 1979

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This report proposes robust detection theoretic approaches to HF signal reception. Most of these approaches require modifying recent robust detection theory results to the specifics of the HF communication problem. One approach recommended is direct and avoids this modification requirement; however, it demands more original effort. The report contains a survey of recent robust detection theory research and discusses the applicability of the research results to HF additive interference models; it concentrates on the known-signal case to highlight the additive interference problem.

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CONTENTS

ON APPROACHES TO ROBUST DETECTION FOR HF COMMUNICATIONS

INTRODUCTION

Simply stated, robust detection capitalizes on all the, a priori, noise model information available to the designer and guarantees a detector performance level against all noise models described by that information. It is suboptimal for each noise model allowed by the a priori information, but generally insensitive to changes from model to model. Suppose you have substantial, but incomplete, knowledge of the additive noise process. Optimal detection would be impossible since it requires complete specification of the noise probability density — you probably would design for a noise that occurs occasionally or not at all. Nonparametric detection would be too conservative since it assumes insufficient knowledge for limiting the noise probability density functions to a class characterizable by a finite number of parameters — you would protect against noise that you know will not occur. For example, a robust detector for nearly-Gaussian noise ([16]) would out-perform a nonparametric detector (e.g., a sign detector) in that noise environment and would be slightly out-performed by the optimal detector in Gaussian noise. Conceptually, robust detection is a game-theoretic ([10], [20]), yet pragmatic, approach to detecting and distinguishing signals in an incompletely-known noise environment.

Consider the robust detection problem as a minimax or game theory problem: Interpret the robust (minimax) detector as the best detector strategy (player 1) against any of nature's (player 2) possible noise model choices within the a priori class of choices. More abstractly, consider the following:

Note: Manuscript submitted November 2, 1978.

PROBLEM: Determine the detector ϕ^o and least-favorable interference probability density vector $\overline{f}^o = (f_0^o, f_1^o, \ldots, f_{M-1}^o)$ for the M-ary hypothesis testing problem,

$$H_i: s_i(t) + w_i(t) \qquad i = 0, \ldots, M-1, \; 0 \leq t \leq T,$$

such that

$$C(\phi^o, \overline{f}^o) = \min_{\phi \in D} \max_{f \in F} C(\phi, \overline{f})$$

where $C(\phi, \overline{f})$ is the cost incurred by the detector ϕ when the interference is defined by $\overline{f}$, D is the class of admissible detectors, and F is the class of interference probability density vectors.

The robust (minimax) detector can be interpreted as the best (with respect to C) choice from D for a <u>class</u> of possible descriptions F. Thus, the optimal ([11], [21], [24], [28]) and the nonparametric ([4], [27]) detection problems are the opposite extremes (F contains one vector and F contains all possible density vectors, respectively) of the robust detection problem and can, in fact, be encompassed by it.

This report proposes robust detection approaches to HF signal reception. Most of these approaches require modifying recent robust detection theory results to the specifics of the HF communication problem. One approach recommended is direct and avoids this modification; however, it demands more original effort. The report contains a survey of recent robust detection theory research and discusses the applicability of the research results to HF additive interference sources and models; it concentrates on the known-signal case to highlight the additive interference problem. Remember, the unknown-signal case, which would be appropriate for treating fading and multipath effects, is essentially the known-signal case either appropriately conditioned (probabilistically) on a <u>random</u> signal variable or assigned to some least favorable known-signal value. A subsequent report will treat the unknown-signal case using this probabilistic conditioning technique.

RECENT RESULTS IN ROBUST DETECTION THEORY

General Contaminated Noise Model — Hypothesis Dependent

The current interest in robust detection began with the work by Huber ([13]) on robust probability ratio tests. He was searching for ratio tests that were insensitive to a few bad observations (extremity data points): so that those few bad observations could not override the majority decision. A general binary-hypothesis independent-measurements model was assumed with different contaminated-nominal sample distributions for each hypothesis; in other words, the actual distributions were known up to some neighborhood of the nominals. The class of possible distribution measures for each hypothesis was given by

$$P_i = \{Q_i | Q_i = (1 - \varepsilon_i)P_i + \varepsilon_i H_i,\ H_i \varepsilon H\} \qquad i = 0, 1$$

where $0 \leq \varepsilon_i \leq 1$ are fixed numbers, and H denotes the class of all probability measures on some given measurable space. This formulation allowed hypothesis-dependent contamination and, consequently, permits interference generated by an intelligent adversary.

Huber obtained the worst-case distribution functions and robust (minimax) ratio tests with respect to three standard performance measures. The performance measures are based on a risk function $(R(Q_i, \phi)$ and given as

i) $\max_i R(Q_i, \phi)$

ii) $R(Q_i, \phi)$ subject to $R(Q_0, \phi) \leq \alpha$ (Neyman-Pearson)

iii) $\lambda_0 R(Q_0, \phi) + \lambda_1 R(Q_1, \phi)$ (Bayes)

His results, which are valid for both fixed sample size and sequential problems, require ε_i small enough to prevent overlapping of the distribution classes (preventing one distribution for both hypotheses). The worst-case (least-favorable) distribution pairs (Q_0, Q_1) generate probability ratios that are censored versions of the nominal probability ratios:

$$q_1(x)/q_0(x) = \begin{cases} bc' & p_1(x)/p_0(x) \le c' \\ bp_1(x)/p_0(x) & c' < p_1(x)/p_0(x) < c'' \\ bc'' & p_1(x)/p_0(x) \ge c'' \end{cases}$$

where $0 \le c' < c'' \le \infty$ are chosen so that q_1 and q_0 are probability densities, and $b = (1-\varepsilon_1)/(1-\varepsilon_0)$. Robust ratio tests ϕ^o are acquired via the Neyman-Pearson Lemma, and the above probability ratios. Thus the robust ratio tests for these performance measures are censored versions of the optimal ratio tests for the nominal densities — an asymmetrical soft-limiter is placed within the optimal detector for the nominal densities. Huber also obtains equivalent results for the class of possible distribution measures described by

$$P_i = \{Q_i | \; ||Q_i - P_i|| \le \varepsilon\} \qquad i = 0, 1$$

where $|| \cdot ||$ denotes total variation.

Contaminated Gaussian Noise Model - Hypothesis Independent

Martin and Schwartz ([16]) modified Huber's results and applied them to the contaminated-normal distribution model for the fixed-sample size, signal detection problem. They sought robust detectors of known signals (possibly time-varying) in nearly-Gaussian i.i.d. noise. The possible noise distributions were assumed to be of the mixture model form

$$F(x) = (1-\varepsilon)\Phi(x) + \varepsilon H(x) \qquad \text{for } 0 \le \varepsilon < 1,$$

where Φ is the unit normal distribution, H is an arbitrary distribution, and ε is small. Both regular and small-signal cases were treated. Some attention was given to the incoherent signal detection problem.

For the nearly-Gaussian noise model $x_i = \theta s_i + n_i$, $i = 1, 2, \ldots, N$, Martin and Schwartz found a time-varying correlator-limiter as the robust (minimax) solution for given ε and $\theta > \theta_\varepsilon/\min_i|s_i|$ with respect to

the Neyman-Pearson performance measure. Explicit expressions for the worst-case (least favorable) densities were obtained. Table 1 displays representative values of ε and θ_ε. Note, the minimum received signal-to-noise ratio (SNR) can be derived from θ_ε via the equation $(SNR)_{min} = 20 \log\left(\frac{s_i\theta_\varepsilon}{n_i}\right)$. For a constant signal ($s_i = c$), the minimax solution is a limiter-detector since correlation is unnecessary.

For the small-signal problem with nearly-Gaussian white noise, the limiter-correlator was found the asymptotically-robust solution to the local (small-signal) Neyman-Pearson problem. Explicit expressions for the worst-case symmetric densities were obtained. This result is valid for false-alarm probability $\alpha \geq \alpha(\varepsilon)$, depending on given ε, and for symmetric contamination densities satisfying a regularity condition. It is valid also for a sample size N = 1 and all $0 \leq \alpha \leq 0.5$. The authors conjectured the validity of this result for a restricted range of α, which depends on N. Table 2 gives typical values of ε and $\alpha(\varepsilon)$. The limiter in this case is a symmetrical soft-limiter with break points K (see table 2). Again with constant signal, the resulting robust detector is the limiter detector.

Martin and Schwartz applied the soft limiter to the envelope sum detector which is the optimal small-signal detector for an incoherent detection of a pulse train. The resulting limiter-envelope sum detector was shown to exhibit (for the examples chosen) the same degree of robustness in terms of asymptotic relative efficiency (ARE) as does the limiter-correlator.

Table 1

$\Phi[\tfrac{1}{2}\theta_\varepsilon] = \tfrac{1}{2}(1-\varepsilon)^{-1}$

ε	θ_ε	20 log θ_ε
0.01	0.025	-32.04 (dB)
0.012	0.03	-30.46
0.02	0.05	-26.02
0.04	0.10	-20.0
0.055	0.15	-16.49
0.06	0.16	-15.92
0.10	0.28	-11.06
0.138	0.40	-7.96

Table 2

ε	$\alpha(\varepsilon)$	K
0.01	0.163	1.95
0.02	0.166	1.72
0.05	0.174	1.40
0.10	0.184	1.14

Contaminated Non-Gaussian Noise Model — Hypothesis Independent

Kassam and Thomas ([14]) generalized the results of Martin and Schwartz for asymptotically-robust local detection of a known time-varying signal by using a contaminated i.i.d. noise model with a non-Gaussian nominal. The results are shown valid for: nominal density functions that are symmetric, strongly unimodal on its support, and have an absolutely continuous derivative within its support; contamination density functions that are bounded and symmetric; and false alarm probabilities $\alpha \geq \alpha(r_m)$, where r_m is a parameter depending on ε. Kassam and Thomas used the methods of Martin and Schwartz and Huber to obtain the worst-case noise density and the asymptotically-robust local detector that is the optimal local detector for that density. The parameter r_m depends also on the nonlinearity function (ZNL) of the optimal local detector.

The results were applied explicitly to zero-mean, generalized-Gaussian nominal densities that are parameterized by their rates of exponential decay.

For this class of nominal densities, $\alpha(r_m)$, for a Gaussian nominal, is a lower bound on α than the lower bound obtained by Martin and Schwartz (see table 3). The general asymptotically-robust local detector solution for this class of nominal densities corroborates the limiter-correlator detector for contaminated Gaussian noise and implies the robustness of the sign detector for contaminated double-exponential noise. For weaker assumptions on the contamination densities (zero median and continuous at the origin), the authors show the sign detector to be the asymptotically robust local detector for all $0 \leq \alpha < 1$.

Table 3

(Gaussian nominal)

ε	$\alpha(r_m)$
0.02	0.103
0.04	0.093
0.06	0.086
0.08	0.081
0.10	0.076

Non-Contaminated-Noise Model — Hypothesis Independent

El-Sawy and VandeLinde ([8], [9]) used a non-contamination (mixture) model for the noise: to eliminate the limitations of earlier work caused by small ε. They used also a restricted class of detectors to reduce the number of detectors considered for their asymptotically-robust detection problem. The binary hypothesis model for this problem takes the form:

$$H_0: x_i = w_i$$

$$H_1: x_i = \theta_1 s_i + w_i, \; i = 1, \ldots, N$$

where $\{x_i\}$ is the sequence of observations, $\theta_1 > 0$, $\{s_i\}$ is a sequence of known constants with $|s_i| < \infty$,

$$\lim_{N\to\infty} N^{-1}\sum_{i=1}^{N} s_i^2 = c,$$

and $\{w_i\}$ is a sequence of i.i.d. random variables with a symmetric density f. A generalized version of Huber's ([12]) M-estimate of location was introduced via a class of functions $\mathcal{C}$ satisfying certain smoothness assumptions. The generalized M-estimates θ_N — the admissible test statistics — are defined as the values of θ which minimize

$$\sum_{i=1}^{N} L(x_i - \theta s_i),\ L \in \mathcal{C}$$

The restricted class of detectors $d \in D$ considered are just the threshold tests obtained from the generalized M-estimate θ_n test statistics. The authors solved a constrained asymptotic (in N) maximin problem with respect to the power function $\beta_d(\theta,|f)$. They derived the saddle-point pair (d_{L^*}, f^*), where f^* is the minimum Fisher-information symmetric density and d_{L^*} is a threshold-test detector for the generalized M-estimate test statistic derived from $L^* = -\log f^*$. The threshold is chosen to yield the desired false alarm rate $\alpha = \beta_d(0|f)$. Equivalent results are obtained for the local detection problem (subject to a lowerbound on α) and for the detection problem (derived with $\{s_i\}$ constant) with a class of p-point noise distributions defined as

$$\{f: \int_{-a}^{a} f(x)dx = p,\ f \text{ symmetric}\}.$$

El-Sawy and VandeLinde claimed also that the results may be used also to solve other hypothesis testing problems, e.g., detection problems for the minimum probability-of-error criterion.

A Robust Detection Problem for HF Communications

The robust detection problem for HF communications is derived from the general robust detection problem by being more specific about the cost function and the HF-nature of the model variables. Using a generalized

binary hypothesis model with the probability-of-error cost function, the robust detection problem for HF communications becomes one of designing a robust test for:

$$H_i: x(t) = s_i(t) + w_i(t), \; i = 0, 1; \; 0 \leq t \leq T$$

where $s_i(t)$, with $|s_i(t)| < \infty$, is the transmitted waveform after the channel filtering effects (front-end filters, channel attenuation, multipath, etc.) and $w_i(t)$ is the additive interference process, with $|w_i(t)| < \infty$, generated by an additive combination of atmospheric noise, thermal noise, man-made noise, other-user interference, and intelligent-adversary interference. In characterizing the additive interference process $w_i(t)$ as an element from a class of possible stochastic processes at the minimum, we must admit the common factors of the accepted (proposed) models for the different types of interference.

Atmospheric noise has been modeled in several ways as an impulsive, non-stationary, correlated random process ([5]-[7], [23], [26]) usually as a combination of a low-level, high probability component whose envelope is Rayleigh distributed and of a high-level, low probability component whose envelope is log-normal distributed. The most recent models ([23], [26]) assume the impulsive interference waveforms are emitted according to the Poisson distribution in time and are superimposed on an additive, independent Gaussian background-noise process. These impulsive waveforms may be modeled as the output of time-varying, possibly stochastically described, linear systems excited by Poisson-distributed impulses. A general atmospheric noise model results described as

$$w_A(t) = w_B(t) + w_I(t)$$

where $w_B(t)$ is zero-mean, independent Gaussian process, and

$$w_I(t) = \sum_{i=1}^{N(t)} u_i \int_{-\infty}^{t} h_i(t,\tau)\delta(\tau-\tau_i)d\tau$$

with h_i a possible impulse response, u_i a finite, amplitude random variable, τ_i the point process variable, and $N(t)$ a counting process ([25]). Thermal noise that is white Gaussian and man-made noise that is impulsive are of secondary concern for HF communications ([6]); nevertheless, they can be adequately handled by this atmospheric noise model. Hence, no further reference will be made to these two sources of interference.

Other-user interference is attributed to randomly-occuring modulated waveforms of various durations and strengths. These waveforms are generally confined to a 3 kHz bandwidth ([6]) — there are some exceptions such as spread-spectrum modulation and coding. The other-user interference waveform $w_{OU}(t)$ also can be modeled as a random process, possibly in the frequency domain, since most available information is from spectra measurements. For example, consider the random point-process model of figure 1.

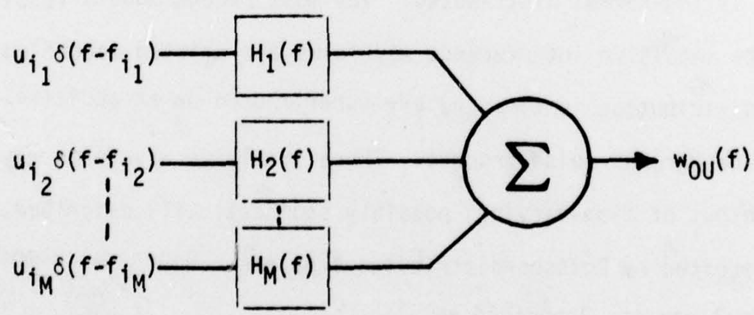

Figure 1

This is a frequency domain description of other-user interference given by

$$w_{OU}(f) = \sum_{k=1}^{M} \sum_{i_k=1}^{N_k(f)} u_{i_k} \int_{-\infty}^{f} H_k(f-\lambda)\delta(\lambda - f_{i_k})d\lambda,$$

which implies,

$$w_{OU}(t) = \sum_{k=1}^{M} \sum_{i_k=1}^{N_k(f)} u_{i_k} h_k(t) e^{j2\pi f_{i_k} t} .$$

$H_k(f)$ is one of the M possible transmitted transforms, u_{i_k} is the finite, signal-amplitude random variable for the i_kth carrier, f_{i_k} is the i_kth carrier-frequency point-process variable, and $N_k(f)$ is a counting process.

Intelligent-adversary interference (jamming) can take any form within the physical constraints imposed by the interference generating equipment. These constraints usually are just upper bounds on peak and average power, and bounds on bandwidth. This characterization admits stochastic (e.g., wideband Gaussian noise) or purely deterministic (e.g., CW) interference that is ideally signal-waveform dependent for maximum effectiveness. For signal-waveform dependence, the interferer requires knowledge of the waveform transmitted; therefore, we assume the interferer knows, a priori, the <u>set</u> of waveforms transmitted, but not the <u>particular</u> waveform transmitted. With these assumptions, the intelligent-adversary interference can be modeled as a stochastic process $w_{IA}(t)$ described by some probability density function (pdf), $f(w_{IA}(t), t) \in \mathcal{F}$, where $f \in \mathcal{F}$ implies:

(a) f is a first-order pdf

(b) $E\{w^2(t)\} \leq c_1 < \infty \qquad 0 \leq t \leq T$

(c) $f(x,t) = 0 \qquad \text{for } |x| > c_2,\ 0 \leq t \leq T$

APPLICABILITY OF RECENT RESULTS

The recent results on robust detection theory, all using discrete-time models, are applicable in only a limited sense to the robust detection problem for HF communications as discussed earlier. The main difficulty is the assumption of i.i.d. interference random variables which is necessary to achieve the results primarily based on asymptotic normality or multiplicative probability-ratio test-statistics. The mixture model used in several treatments also presents some difficulty. It assumes a time-sharing of statistical descriptions: one required to occur only a small percentage of (ε) of the time and the other a nominal required to be completely known statistically. Finally, the requirement of symmetrical interference densities for validity of some results is possibly a very restrictive constraint on interference characterization for this problem.

This assumption of i.i.d. interference random variables certainly is not met by sampled values of interference from atmospheric noise or other-user interference; and it may be a costly limitation for intelligent-adversary interference, although that has to be determined. Although techniques for accommodating the non-i.i.d. nature (primarily the dependence) of the HF interference are desirable the independence assumption may be used to determine bounds on detector performance. The general argument is that dependence (correlation) among samples provides additional information for designing counter-strategies, thus, the assumption of independence leads to bounds on performance for the dependent case. In addition, the identically-distributed assumption may not be required for validity results previously mentioned since it is not necessary for asumptotic normality ([1], ch. 8) nor required for the canonical form of the probability ratio tests ([20]).

The mixture models assumed for the interference density require complete statistical knowledge of the nominal density; yet for the HF interference discussed, all of the components — atmospheric noise, other-user interference, and intelligent-adversary interference (when applied) — are not known exactly. Thus, the reported results for this mixture model could not be applied directly to the general HF communication problem as formulated here. You could interpret the mixture model, with its time-sharing implication, as: (1) a high probability ($1-\epsilon$) nominal density describing all but the impulsive noise, and a low probability (ϵ) contaminated density describing the nominal <u>plus</u> the impulsive noise ([17]); or, (2) a high probability nominal including all but the intelligent-adversary interference ([26]) and a low probability contaminated density describing the nominal <u>plus</u> intelligent-adversary interference. In other words, the contaminating interference component and the other noise components would be assumed to occur with an average rate of ϵ. The mixture-model results may be applied to several typical nominal components; the outcome may be instructive. typical densities for atmospheric noise, other-user interference, intelligent-adversary interference, or any combination would be the candidates for the nominals. The robust detectors obtained from the different nominals could then be compared for similarities that could be exploited later. Another difficulty in applying Huber's mixture-model results to the HF problem is the assumption of a signal-dependent interference capability that is not assumed in the HF communication problem formulated here. If possible, it may be instructive to consider detectors for signal-independent, contaminating interference derived from a random or non-ramdom convex combination of the least-favorable signal-dependent densities.

In the problem formulated here for HF communications, the cost function prescribed is the probability of error (Bayes) performance measure. However,

only Huber ([13]) considered this cost function explicitly. El-Sawy and VandeLinde indicated their techniques are applicable to the probability of error cost function. The cost function considered in all of the research discussed is the one appropriate to radar problems, namely, the probability of detection failure (miss probability $1-\beta$). However, since the detectors chosen with respect to this radar cost function are generally required to satisfy a probability of false-alarm (α) constraint (the Neyman-Pearson problem), the solution for the communication (Bayes) problem can be arrived at by solving a series of radar problems. This follows from the fact that the probability of error is a weighted sum of the miss and false-alarm probabilities and, consequently, the minimum probability-of-error detector is also a minimum probability-of-miss detector achieving a particular α ([11], sec. 3.2). Unfortunately, the results reviewed above for the interference-mixture-model Neyman-Pearson problem are valid only for false-alarm probabilities larger than a given lower bound $\alpha(\varepsilon)$ and, therefore, may not be applicable to the HF communication problem formulated above. Unless the Bayes detector can be found among those Neyman-Pearson detectors satisfying the lower bound constraint $\alpha(\varepsilon)$, the results for the mixture model problems cannot be applied. All attention must then be given either to applying the results of Huber and of El-Sawy and VandeLinde or to solving directly the robust detection problem for HF communications.

The requirement of symmetry for the interference densities may be a very restrictive limitation for this HF problem. This symmetry of densities implies zero-mean interference and certainly eliminates other-user interference. Moreover, it is not known at the outset if restricting intelligent-adversary interference to being zero-mean is severly limiting or not for this problem. For example; Blachman ([2], [3] ch. 9) has shown that

zero-mean, white Gaussian noise is the best intelligent-adversary interference for minimizing the channel capacity seen by a communicator; and, it can be shown ([19], [20]) that deterministic signals are the best interference for minimizing the correlation in correlator receivers. Thus, it appears the symmetry requirement possibly eliminates or sharply curtails the two main components of the interference characterized in the HF problem formulated above.

RECOMMENDATIONS FOR APPROACHES

From the review of the recent results on robust detection and the discussion of their applicability to the robust detection for HF communications problem, recommendations can be made for approaching this problem.* The first approach recommended is based on Huber's ([13]) formulation and results. For the second approach, a direct solution attempt is recommended. In both cases, the concern is for the binary detection problem where both signal values are non-zero. In addition, both approaches are based on a discrete-time model in anticipation of digital processing.

Approach 1

In the first approach; the mixture model, the lack of symmetry in the interference densities, and the treatment of the finite sample problem — instead of the asymptotic problem — are the motivating factors for utilizing Huber's results. The mixture model for the interference densities will be used to describe the interference random variables $\{w(t_k)\}$ when impulsive noise (from atmospheric noise or man-made noise) or intelligent-adversary interference are present or absent. (Note, these are two separate problems.) The nominal density will describe an arbitrary density for the interference random variables without either the impulsive noise or intelligent-adversary interference component. The contaminated density, occurring with probability ϵ, will describe the

* We recognize the inappropriateness of the known-signal detection problem model for the complete HF problem ([18], [21], [22], [24], [28]). However, by treating the easier robust detection problem for known signals first, we gain valuable insight as well as possible solutions to the robust detection problem for unknown signals.

interference random variables containing the impulsive noise or the intelligent-adversary interference component. The interference random variables will be assumed independent, but not identically distributed.

For this formulation, Huber's results extended to time-varying signals ([16]) and non-identically-distributed random variables can be applied directly. The minimax detector obtained in this first step and based on a probability ratio test will be dependent on the arbitrary nominal specification. The next step will be to optimize this minimax detector over the class or appropriate subclasses of possible nominal densities. The resulting minimax detector obtained in the second step will have the robust properties desired for the HF communication problem. Steps 1 and 2 can be abstractly stated as:

Step 1: Find the detector ϕ_ε^o and least-favorable mixture interference-density $f_\varepsilon^o = (1-\varepsilon)f_o + \varepsilon f_I^o$, for a given ε and nominal f_o, satisfying

$$C[\phi_\varepsilon^o(f_o), f_\varepsilon^o(f_o)] = \min_{\phi_\varepsilon \in D} \max_{f_\varepsilon \in F_\varepsilon} C[\phi, f_\varepsilon]$$

where $C[\phi, f_\varepsilon]$ is the probability of error using detector ϕ and mixture interference density f_ε, D is the class of detectors based on probability ratio tests, and $F_\varepsilon = \{f_\varepsilon | f_\varepsilon = (1-\varepsilon)f_o + \varepsilon f_I;\ f_I \text{ any density}\}$.

Step 2: Find the detector ϕ_ε^{oo} and least-favorable mixture interference-density f_ε^{oo}, for a given ε and class of possible nominal densities F_o, satisfying

$$C[\phi_\varepsilon^{oo}, f_\varepsilon^{oo}] = \min_{\phi_\varepsilon^o \in D_\varepsilon^o} \max_{f_\varepsilon^o \in F_\varepsilon^o} C[\phi_\varepsilon^o, f_\varepsilon^o]$$

where $C[\phi_\varepsilon^o, f_\varepsilon^o]$ is the probability of error using detector ϕ_ε^o (minimax for given ε and some $f_o \in F_o$) and mixture interference density f_ε^o (least favorable for given ε and some $f_o \in F_o$), $D_\varepsilon^o = \{\phi_\varepsilon^o | \phi_\varepsilon^o$ is a minimax detector with respect to F_ε for given ε and some $f_o \in F_o\}$, and $F_\varepsilon^o = \{f_\varepsilon^o | f_\varepsilon^o = (1-\varepsilon)f_o + F_I^o; \forall f_o \in F_o\}$.

After evaluation of this robust detector, additional methods for improving performance may become apparent. For example, including an adaptivity capability for following the time occurrence of the contaminated density would be advantageous presumably since the detector design depends explicitly on the occurence rate ε. As an alternative, keeping the detector non-adaptive and seeking a detector that would be robust also on a range of ε may be desirable. This alternative method could constitute step three, abstractly stated as:

Step 3: Find the detector $\phi^{oo}_{\varepsilon^o}$, and least-favorable mixture interference-density $f^{oo}_{\varepsilon^o}$, for a given class of possible nominal densities F_o and given range $0 \leq \varepsilon \leq \varepsilon_{max}$ satisfying

$$C[\phi^{oo}_{\varepsilon^o_1}, f^{oo}_{\varepsilon^o_2}] = \min_{\phi^{oo}_{\varepsilon_1} \in D^o_{max}} \max_{f^{oo}_{\varepsilon_2} \in F^o_{max}} C[\phi^{oo}_{\varepsilon_1}, f^{oo}_{\varepsilon_2}]$$

where $C[\phi^{oo}_{\varepsilon_1}, f^{oo}_{\varepsilon_2}]$ is the probability of error using detector $\phi^{oo}_{\varepsilon_1}$ (robust for some $F^o_{\varepsilon_1}$ with $0 \leq \varepsilon_1 \leq \varepsilon_{max}$) and mixture interference density $f^{oo}_{\varepsilon_2}$ (least favorable for some $D^o_{\varepsilon_2}$ with $0 \leq \varepsilon_2 \leq \varepsilon_{max}$), $D^o_{max} = \{\phi^{oo}_{\varepsilon_1} | 0 \leq \varepsilon_1 \leq \varepsilon_{max}\}$, and $F^o_{max} = \{f^{oo}_{\varepsilon_2} | 0 \leq \varepsilon_2 \leq \varepsilon_{max}\}$.

This first approach should produce promising results to apply to the HF communication problem. A major weakness is assuming independence for the interference random variables. Consequently, the performance of this detector for dependent interference random variables should be evaluated, probably by appropriate tests and simulations; analysis may be possible, however, for some forms of independency, e.g., state variable models or Markovian requirements.

Approach 2

For the second approach, an attempt should be made to solve directly the formulated problem using generalized Lagrange-multiplier theory on normed linear spaces ([15]). By modeling the possible, interference random-sequence densities as members of a subset of a space of multivariate functions, the property of dependency among the random sequences components can be treated in a convenient manner: you are not required to specify a dependency relationship for the subset of multivariate functions. Characterizing this subset will, however, require the statistical descriptions of the interference components $w_A(t_k)$, $w_{OU}(t_k)$, and $w_{IA}(t_k)$ discussed in the HF communication problem formulation. This approach can be expressed more concisely as:

Step 1: Find the detector ϕ^o and least-favorable multivariate density f^o satisfying

$$C[\phi^o, f^o] = \min_{\phi \in D} \max_{f \in F} C[\phi, f]$$

where $C[\phi, f]$ is the probability of error using detector ϕ and interference multivariate density f, D is the class of detectors based on probability ratio tests, and F is the subset of admissible multivariate functions for which $f = f(\overline{w})$ F implies f is a probability density for the interference random sequence $\overline{w} = \overline{w}_A + \overline{w}_{OU} + \overline{w}_{IA} = (w(t_1), \ldots\ldots, w(t_N))$.

Characterizing the subset of admissible densities F will probably be the most difficult aspect of this approach since $f(\overline{w}) \in F$ results from convolving the individual probability densities for the background noise $\overline{w}_B$, impulsive noise $\overline{w}_I$, other-user $\overline{w}_{OU}$, and intelligent-adversary $\overline{w}_{IA}$ interference components. The noise model of Spaulding and Middleton [26] may be applicable here. In choosing the normed linear space $\mathcal{F}$, for which $F \subset \mathcal{F}$, the most likely choice is $L_1[\Omega^N]$, the normed linear space of absolutely integrable functions on the

domain Ω^N ([15]). In this space, all non-negative functions that have a norm of one represent probability densities.

Another difficult aspect of this approach may be in specifying exactly the least-favorable density after applying the Lagrange multiplier theory. The theory provides necessary — but generally not sufficient — specifications defining the class of least-favorable density candidates; also it provides the validation method for identifying the least-favorable density. In many instances the class of density candidates are infinite and the least-favorable density may be quite time-consuming to find without some assistance from helpful hints and luck. One technique for gaining this investigative assistance (and other insights) for multistage problems is to solve a simpler corresponding single-stage problem. Another technique is to solve a more general problem for which the desired problem is a special or restricted case. These techniques will be used in this second approach.

This second approach is more ambitious than the first approach and should yield more realistic results. The performance of the derived robust detectors in the actual HF environment depends on how closely the description of the subset of admissible densities F models the actual environment. If the direct approach appears too formidable in this regard, this closeness of the model to the actual environment may be accomplished in successive steps. The first step (Step 1a) could be equivalent to Step 1 above with F′ defined as a larger subset of $\mathcal{F}$: for example, the subset containing all probability densities. A valid corresponding second step (Step 1b) could be a sensitivity investigation of the Step 1a robust detector subjected to deviations from the corresponding least-favorable density. These deviations would be typical densities from the class F defined in Step 1 above. Hopefully, some least-favorable deviation could be found in this step. Another valid corresponding second step (Step 1b′)

would be equivalent to Step 1a with F' replaced with a proper subset $F'' \subset F'$, where $F \subset F''$. This series of nested problems (Step 1a, Step 1b$'$, . . .) can continue in principle until the problem Step 1 is reached or approximated closely enough for practical purposes. An advantage of this nested problem approach, hopefully for little effort in characterizing F', F'', . . . , is the considerable insight likely gained as to how the corresponding robust detectors $\phi^{o\prime}, \phi^{o\prime\prime}$, . . . , approach the robust detector ϕ^{o} of Step 1.

REFERENCES

1. R. B. Ash, Real Analysis and Probability. New York: Academic Press, 1972.

2. N. M. Blachman, "Communication as a Game", IRE Wescon Conv. Rec.: Part 2, pp. 61-66, 1957.

3. N. M. Blachman, Noise and Its Effect on Communication. New York: McGraw-Hill, 1966.

4. J. W. Carlyle, "Nonparametric Methods in Detection Theory", in Communications Theory, A. V. Balakrishnan, Ed.. New York: McGraw-Hill, 1968, Ch. 8.

5. R. M. Coon, E. C. Bolton, and W. E. Bensema, "A Simulator for HF Atmospheric Radio Noise", U.S. Dept. Commerce, ESSA Tech. Rep. ERL 128-ITS 90, July 1969 (U.S. Printing Office, Wash., DC 20402).

6. P. J. Crepeau, "Topics in Naval Telecommunications Media Analysis", U.S. Naval Research Laboratory, NRL Rep. 8080, 31 Dec 1976, (Natl. Technical Inform. Service, Dept. of Commerce, Springfield, VA 22161).

7. R. T. Disney and A. D. Spaulding, "Amplitude and Time Statistics of Atmospheric and Man-Made Noise", U.S. Dept. of Commerce, ESSA Tech. Rep. ERL 150-ITS 98, Feb. 1970 (U.S. Printing Office, Wash., DC 20402).

8. A. H. El-Sawy and V. D. VandeLinde, "Robust Detection of Signals in Symmetrically Distributed Noise", Proc. 1976 Conf. Inform. Sci. and Sys., Baltimore, MD, pp. 487-488, Mar. 1976.

9. A. H. El-Sawy and V. D. VandeLinde, "Robust Detection of Known Signals", The Johns Hopkins University, Baltimore, MD, Tech. Rep. No. EE 76-5, 1976.

10. T. S. Ferguson, Mathematical Statistics: A Decision Theoretic Approach. New York: Academic Press, 1967.

11. J. C. Hancock and P. A. Wintz, Signal Detection Theory. New York: McGraw-Hill, 1966.

12. P. J. Huber, "Robust Estimation of a Location Parameter", Ann. Math. Statist., Vol. 35, pp. 73-101, Mar. 1964.

13. P. J. Huber, "A Robust Version of the Probability Ratio Test", Ann. Math. Statist., Vol. 36, pp. 1753-1758, Dec. 1965.

14. S. A. Kassam and J. B. Thomas, "Asymptotically Robust Detection of a Known Signal in Contaminated Non-Gaussian Noise", IEEE Trans. Inform. Thy., Vol. IT-22, No. 1, pp. 22-26, Jan. 1976.

15. D. G. Luenberger, Optimization by Vector Space Methods. New York: John Wiley and Sons, 1969.

16. R. D. Martin and S. C. Schwartz, "Robust Detection of a Known Signal in Nearly Gaussian Noise", IEEE Trans. Inform. Thy., Vol. IT-17, No. 1, pp. 50-56, Jan. 1971.

17. J. H. Miller and J. B. Thomas, "The Detection of Signals in Impulsive Noise Modeled as a Mixture Process", IEEE Trans. Commun., Vol. COM-24, No. 5, pp. 559-563, May 1976.

18. J. W. Modestino, Private Communication.

19. J. M. Morris, "On Worst-Case Interference for M-ary Signaling and Correlation Receivers", in preparation.

20. W. L. Root, "Communications Through Unspecified Additive Noise", Information and Control, Vol. 4, No. 1, pp. 15-29, Mar. 1961.

21. W. L. Root, "An Introduction to the Theory of the Detection of Signals in Noise", Proc. IEEE, Vol. 58, No. 5, pp. 610-623, May 1970.

22. W. L. Root, Private Communication.

23. B. Sankur, "Modelling and Analysis of Impulsive Noise", Proc. NATO Advanced Study Institute on Commun. Sys. and Random Process Thy., Edited by J. K. Skwirzynski, Leyden, Netherlands: A. W. Sijthoff, 1977.

24. I. Selin, Detection Theory. Princeton, NJ: Princeton University Press, 1965.

25. D. L. Snyder, Random Point Processes. New York: John Wiley and Sons, 1975.

26. A. D. Spaulding and D. Middleton, "Optimum Reception in an Impulsive Interference Environment - Parts I and II", IEEE Trans. Commun., Vol. COM-25, No. 9, pp. 910-934, Sept. 1977.

27. J. B. Thomas, "Nonparametric Detection", Proc. IEEE, Vol. 58, No. 5, pp. 623-631, May 1970.

28. A. D. Whalen, Detection of Signals in Noise. New York: Academic Press, 1971.

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